

Let Γ be a lattice in $SO_0(3,1)$, and let $M = \Gamma \backslash \mathbb{H}^3$ be the corresponding hyperbolic 3-manifold. It is well-known that under certain geometric hypotheses (for instance, when M has a closed, embedded, totally-geodesic surface), the canonical flat conformal structure on M can be deformed. Equivalently, the lattice Γ admits non-trivial deformations into $SO_0(4,1)$.

Menasco and Reid have shown that many hyperbolic knot complements fail to contain a closed, embedded, totally-geodesic surface, and conjecture that this is the case for all hyperbolic knot complements. Nevertheless, we have found a kind of deformation which works for some well-known families of hyperbolic knots and links, and is distinct from all previously known deformations arising from totally-geodesic surfaces. Indeed, the smallest of these knot complements contains no closed incompressible surface at all.