OUTLINE OF "LORENTZ SPACETIMES OF CONSTANT CURVATURE" BY GEOFF MESS

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(1) Introduction (§1). The goal of the paper is to classify constant curvature $2 + 1$-dimensional domains of dependence with closed Cauchy surfaces. The strategy is the same in each case (flat, de Sitter, and anti-de Sitter): construct a certain class of maximal "standard spacetimes" of a given topological type and then prove that an arbitrary example embeds in a standard spacetime.

(2) The Flat Case (§2 – §5)

  (a) Fuchsian holonomy (§2). The main result is Proposition 1, that the linear holonomy is discrete, using Goldman's theorem. There is a terse exposition of Milnor-Wood inequality and Goldman's theorem. Note there is a second proof of discreteness in §8 (Prop. 24).

  (b) Realization of holonomy (§3). The main result is Proposition 3, which is an application of the holonomy theorem to say that a small affine deformation of a given linear representation is realized by a future complete flat spacetime.

  (c) Standard spacetimes (§4). Proposition 4 shows that the developing image of a spacelike slice is an embedding and that the time coordinate is proper. The standard spacetimes are defined and Proposition 6 shows that arbitrary spacelike slices embed together with a standard spacetime. The second half of this section handles, more or less completely, the genus 1 case.

  (d) Domains of dependence and geodesic laminations (§5). This section contains the main theorem in the flat case; essentially that domains of dependence are in one-one correspondence with measured geodesic laminations. First some basic causality; domains of dependence are defined and Proposition 11 describes the structure of the causal horizon. Proposition 12 is one half of the main theorem; given a hyperbolic surface and measured geodesic lamination, there is a corresponding flat spacetime. The proof is long and contains a lot of implicit information on the structure of these examples. Proposition 13 is the other half; that an arbitrary domain of dependence determines a measured geodesic lamination, inverse to the correspondence in Prop. 12. The section concludes with Propositions 14 and 15 which show that the group action on the causal horizon is complicated and uses this to characterize when domains of dependence embed in larger spacetimes.

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(3) The de Sitter Case (§6). Pages 20-24, leading up to Proposition 16, show how to create de Sitter domains of dependence from a complex projective structure on a closed surface. The converse (that every de Sitter domain of dependence comes from this construction) is not proved in general, but is discussed on p.25, with a proof in $1 + 1$ given as Prop. 17, and the genus 1 case in $2 + 1$ as Prop. 18.

(4) The anti-de Sitter Case (§7). Proposition 19 gives a map from anti-de Sitter domains of dependence to $Teich \times Teich$, using Goldman's Theorem. Proposition 20 proves that this map is onto, by constructing the "standard" anti-de Sitter examples. The next few pages lead up to Proposition 21, which states that every anti-de Sitter domain of dependence embeds in a standard example.This is followed by a discussion and proof of the Earthquake theorem using this setup.

(5) Classification theorems (§8). Proposition 25 is the "Mess theorem" that closed surface groups can't act properly on all of Minkowski space. The proof relies on all of the machinery developed in the flat case. Proposition 26 uses the Carrière ellipsoid argument to show that every flat spacetime with spacelike boundary is a product; this is similar to Proposition 15. Proposition 27 again uses Carrière to prove a similar statement in the anti-de Sitter case.

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