PROBLEM LIST FOR THE "NEW TECHNIQUES IN LORENTZ MANIFOLDS" WORKSHOP

(1) (Labourie) Mess shows that compact oriented orthochronous $2+1$ AdS spacetime with non-empty spacelike boundary S a product $S \times [0,1]$ and embeds in a domain of dependence. Is it possible to construct a singular AdS manifold with more than two ends, say by branching on a spacelike geodesic in a domain of dependence?

(2) (Scannell) Generalize the "no topology change" theorem of Mess noted above to all constant curvature $3+1$ spacetimes. Or (even better) characterize when a constant curvature $3+1$ maximal domain of dependence embeds in a larger constant curvature spacetime.

(3) (Schlenker) Let M be a compact AdS cone manifold with m singular curves. Given real numbers $\dot{\alpha_1}, \dots \dot{\alpha_m}$, is there a first order deformation of the AdS structure inducing these derivatives of the cone angles? This is related to the following problem, posed by Mess.

(4) (Mess) Let $\rho = (\rho_L, \rho_R)$ be the representation of the fundamental group of a closed surface into $PSL(2,\mathbb{R}) \times PSL(2,\mathbb{R})$ corresponding to an AdS domain of dependence.

   (a) Is ρ determined by the two measured laminations on the boundary of the "convex hull"?
   (b) Is ρ determined by the hyperbolic structure on the future boundary of the convex hull together with the measured lamination on the past boundary?
   (c) Is ρ determined by ρ_L together with the hyperbolic structure on one of the boundary components of the convex hull?

   These are analogous to well-known questions about the parameterization of quasi-Fuchsian space by the pair of conformal structures at infinity, and how these relate to the bending laminations and hyperbolic structures on the convex hull boundary.

(5) (Barbot) Let M_0 be a globally hyperbolic static AdS spacetime with closed spacelike slices and consider $v_0 = vol(M_0)$. Is the volume of a non-static AdS spacetime of the same topological type less than or equal to v_0?

Date: November 10, 2004.

(6) (Schlenker) Is the volume of the convex core of a $2+1$ AdS domain of dependence strictly concave as the bending lamination varies? This question, and Barbot's question above, can be thought of as refinements of the following question posed by Mess in his preprint:

(7) (Mess) For a $2+1$ AdS domain of dependence, the volume of the maximal domain of dependence and of the convex core are invariants on $Teich \times Teich$. How do they behave? Are they related, perhaps asymptotically, to invariants of on quasi-Fuchsian space such as the volume of the convex core and the Hausdorff dimension of the limit set?

(8) (Harris) A static complete spacetime is conformal to $(\mathbb{L}^1 \times M)/G = U$ with $G \subset \mathrm{Isom}(M)$ for a Riemannian manifold M. Here $\mu : G \to \mathbb{R}$ is a homomorphism and G acts on $\mathbb{L}^1 \times M$ by $g(t,x) = (t + \mu(g), g \cdot x)$. Does $\hat{\partial}(U)$ depend on μ?

(9) (Goldman) Let M be a complete flat $2+1$ spacetime.
   (a) Does M have a fundamental domain bounded by crooked planes?
   (b) Is the interior of M diffeomorphic to a solid handlebody?
   (c) Do there exist natural smooth approximations of crooked planes?
   (d) (Properness conjecture). It is known that if an affine deformation of a Fuchsian group acts properly, then the value of the Margulis invariant is everywhere positive or everywhere negative. Is the converse true?

(10) (Goldman) Extend crooked planes to surfaces in AdS space. Are there conformally invariant surfaces that could be used as boundaries of fundamental domains of AdS spacetimes?

(11) (Abels)
   (a) Auslander Conjecture: Is every affine crystallographic group (i.e. discrete, cocompact subgroup of $\mathrm{Aff}(\mathbb{R}^n)$ acting properly) virtually solvable?
   (b) Are there properly discontinuous affine groups (not necessarily cocompact) that are neither virtually polycyclic nor virtually free?

(12) (Scannell) Characterize closed hyperbolic 3-manifolds which admit affine deformations into $\mathrm{Isom}(\mathbb{R}^4_1)$. Do they always admit quasi-Fuchsian deformations into $\mathrm{Isom}(\mathbb{H}^4)$?

(13) (Brenner) Let Γ be a discrete subgroup of the complex-affine group $\mathrm{Aff}_3(\mathbb{C})$ whose action on $\mathbb{C}^3$ is properly discontinuous and co-compact. That is Γ

is a *complex affine crystallographic group of dimension* 3. Assume in addition that Γ is virtually nilpotent. Theorem: there is a finite-index subgroup Γ_0 of Γ such that Γ_0 contains two translations t_1, t_2 linearly independent over $\mathbb{R}$. Further, the plane $\mathbb{R}\langle t_1, t_2\rangle$ spanned by t_1, t_2 is a complex plane, in the sense that this plane equals $\mathbb{C}\langle t_1\rangle$.

(a) Is there a "conceptual" proof of this theorem, meaning one not dependent on the detailed classification of similar structures in dimension 2?

(b) What does the theorem imply about all complex, complete, affinely flat, compact manifolds M in dimension 3?

(c) Which if any of the manifolds mentioned in (b) have Lorentzian or other pseudo-Riemannian metrics and do these satisfy any stronger forms of the theorem?